White Rose Maths

Fractions

Overview

Small Steps

Notes for 2020/21

- What is a fraction? ®
- Equivalent fractions (1) ®
- Equivalent fractions
- Fractions greater than 1 ®
- Improper fractions to mixed numbers
- Mixed numbers to improper fractions
- Number sequences
- Compare and order fractions less than 1
- Compare and order fractions greater than 1
- Add and subtract fractions
- Add fractions within 1
- Add 3 or more fractions
- Add fractions

Children will need to look at different representations of fractions to expose any misconceptions.

They can then move onto a practical exploration of equivalent fractions by folding paper before comparing fractions with drawings and diagrams in these first recap steps.

Year 5 is the first time children explore improper fractions in depth so we have added a recap step from Year 4 where children add fractions to a total greater than one whole.

Overview

Small Steps

Notes for 2020/21

- Add mixed numbers
- Subtract fractions
- Subtract mixed numbers
- Subtract – breaking the whole
- Subtract 2 mixed numbers
- Multiply unit fractions by an integer
- Multiply non-unit fractions by an integer
- Multiply mixed numbers by integers
- Calculate fractions of a quantity ®
- Fraction of an amount
- Using fractions as operators

As children progress through the small steps they use different representations to support their understanding of the abstract.

Before exploring fractions of an amount it may be useful to recap the Year 4 content with practical equipment and pictorial representations to help them see the relationships between the fraction and the whole.

What is a Fraction?

Notes and Guidance

Children explore fractions in different representations, for example, fractions of shapes, quantities and fractions on a number line.

They explore and recap the meaning of numerator and denominator, non-unit and unit fractions.

Mathematical Talk

How can we sort the fraction cards?

What fraction does each one represent?

Could some cards represent more than one fraction?

Is $\frac{1.5}{3}$ an example of a non-unit fraction? Why?

Using Cuisenaire, how many white rods are equal to an orange rod? How does this help us work out what fraction the white rod represents?

Varied Fluency

Here are 9 cards.
Sort the cards into different groups.
Can you explain how you made your decision?
Can you sort the cards in a different way?
Can you explain how your partner has sorted the cards?
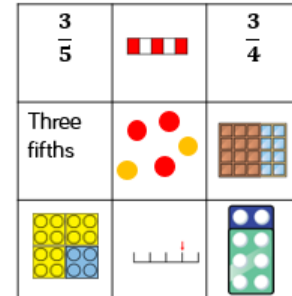
Complete the Frayer model to describe a unit fraction.

Can you use the model to describe the following terms?

Non-unit fraction   Numerator   Denominator

Definition	Characteristics
Examples	Non-examples
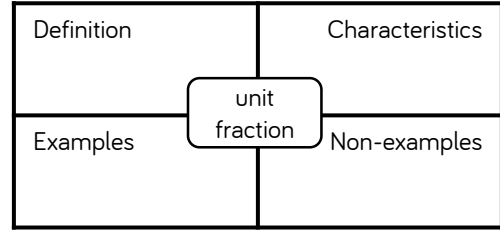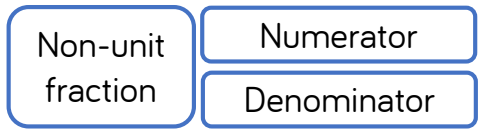
unit fraction

Use Cuisenaire rods.
If the orange rod is one whole, what fraction is represented by:
- The white rod   - The red rod
- The yellow rod   - The brown rod

Choose a different rod to represent one whole; what do the other rods represent now?

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What is a Fraction?

Reasoning and Problem Solving

R

Always, Sometimes, Never?

Alex says,

> If I split a shape into 4 parts, I have split it into quarters.

Explain your answer.

Sometimes

If the shape is not split equally, it will not be in quarters.

Which representations of $\frac{4}{5}$ are incorrect?

$\dfrac{4}{5}$

Explain how you know.

The image of the dogs could represent $\frac{2}{5}$ or $\frac{3}{5}$

The bar model is not divided into equal parts so this does not represent $\frac{4}{5}$

Equivalent Fractions (1)

Notes and Guidance

Children use strip diagrams to investigate and record equivalent fractions.

They start by comparing two fractions before moving on to finding more than one equivalent fraction on a fraction wall.

Mathematical Talk

Look at the equivalent fractions you have found.
What relationship can you see between the numerators and denominators? Are there any patterns?

Can a fraction have more than one equivalent fraction?

Can you use Cuisenaire rods or pattern blocks to investigate equivalent fractions?

Varied Fluency

Use two strips of equal sized paper.
Fold one strip into quarters and the other into eighths.
Place the quarters on top of the eighths and lift up one quarter; how many eighths can you see?
How many eighths are equivalent to one quarter?
Which other equivalent fractions can you find?

Using squared paper, investigate equivalent fractions using equal parts e.g. $\frac{2}{4} = \frac{?}{8}$
Start by drawing a bar 8 squares long.
Underneath, compare the same length bar split into four equal parts.

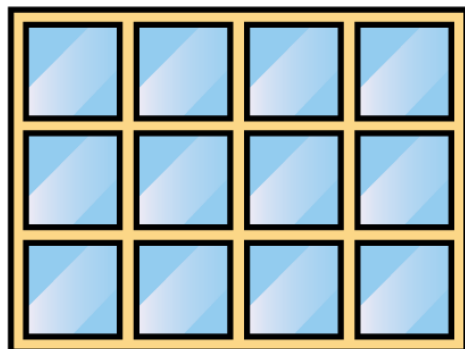How many fractions that are equivalent to one half can you see on the fraction wall?
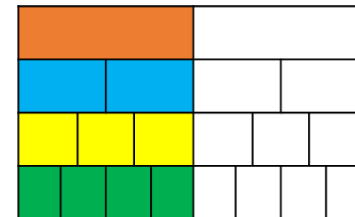
Draw extra rows to show other equivalent fractions.

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Equivalent Fractions (1)

Reasoning and Problem Solving

R

How many equivalent fractions can you see in this picture?

Children can give a variety of possibilities. Examples:

$$\frac{1}{2} = \frac{6}{12} = \frac{3}{6}$$

$$\frac{1}{4} = \frac{3}{12}$$

Eva says,

I know that $\frac{3}{4}$ is equivalent to $\frac{3}{8}$ because the numerators are the same.

Is Eva correct?
Explain why.

Eva is not correct. $\frac{3}{4}$ is equivalent to $\frac{6}{8}$ When the numerators are the same, the larger the denominator, the smaller the fraction.

Ron has two strips of the same sized paper.
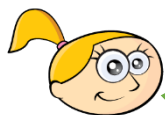He folds the strips into different sized fractions.
He shades in three equal parts on one strip and six equal parts on the other strip.
The shaded areas are equal.

What fractions could he have folded his strips into?

Ron could have folded his strips into sixths and twelfths, quarters and eighths or any other fractions where one of the denominators is double the other.

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Equivalent Fractions

Notes and Guidance

Children explore equivalent fractions using models and concrete representations.

They use models to make the link to multiplication and division. Children then apply the abstract method to find equivalent fractions.

It is important children have the conceptual understanding before moving on to just using an abstract method.

Mathematical Talk

What equivalent fractions can we find by folding the paper? How can we record these?

What is the same and what is different about the numerators and denominators in the equivalent fractions?

How does multiplication and division help us find equivalent fractions? Where can we see this in our model?
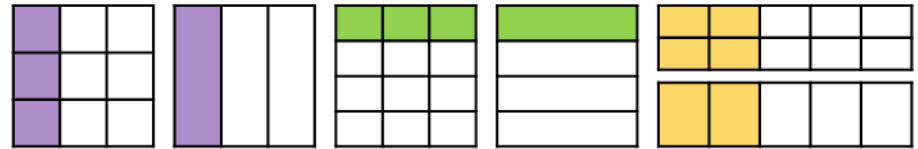
Varied Fluency

Take two pieces of paper the same size.
Fold one piece into two equal pieces.
Fold the other into eight equal pieces.
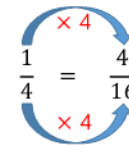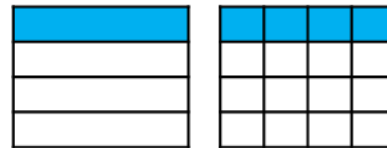What equivalent fractions can you find?
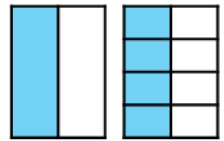
$$\frac{1}{2} = \frac{4}{8}$$

Use the models to write equivalent fractions.

Eva uses the models and her multiplication and division skills to find equivalent fractions.

$$\frac{1}{4} = \frac{4}{16}$$

× 4   × 4

Use this method to find equivalent fractions to $\frac{2}{4}, \frac{3}{4}$ and $\frac{4}{4}$ where the denominator is 16

Eva uses the same approach to find equivalent fractions for these fractions. How will her method change?

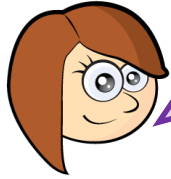$$\frac{4}{12} = \frac{\square}{3}$$ $$\frac{6}{12} = \frac{\square}{4}$$ $$\frac{6}{12} = \frac{\square}{2}$$

Equivalent Fractions

Reasoning and Problem Solving

Rosie says,

To find equivalent fractions, whatever you do to the numerator, you do to the denominator.

Using her method, here are the equivalent fractions Rosie has found for $\frac{4}{8}$

$$\frac{4}{8} = \frac{8}{16} \qquad \frac{4}{8} = \frac{6}{10}$$

$$\frac{4}{8} = \frac{2}{4} \qquad \frac{4}{8} = \frac{1}{5}$$

Are all Rosie's fractions equivalent?
Does Rosie's method work?
Explain your reasons.

$\frac{4}{8} = \frac{1}{5}$ and $\frac{4}{8} = \frac{6}{10}$ are incorrect.

Rosie's method doesn't always work. It works when multiplying or dividing both the numerator or denominator but not when adding or subtracting the same thing to both.

Ron thinks you can only simplify even numbered fractions because you keep on halving the numerator and denominator until you get an odd number.

Do you agree?
Explain your answer.

Ron is wrong. For example $\frac{3}{9}$ can be simplified to $\frac{1}{3}$ and these are all odd numbers.

Here are some fraction cards.
All of the fractions are equivalent.

$$\frac{4}{A} \qquad \frac{B}{C} \qquad \frac{20}{50}$$

A + B = 16
Calculate the value of C.

A = 10
B = 6
C = 15

Fractions Greater than 1

Notes and Guidance

Children use manipulatives and diagrams to show that a fraction can be split into wholes and parts.

Children focus on how many equal parts make a whole dependent on the number of equal parts altogether. This learning will lead on to Year 5 where children learn about improper fractions and mixed numbers.

Mathematical Talk

How many _____ make a whole?

If I have _____ eighths, how many more do I need to make a whole?

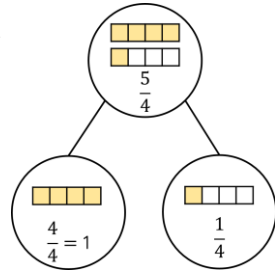What do you notice about the numerator and denominator when a fraction is equivalent to a whole?

Varied Fluency

Ⓡ

Complete the part-whole models and sentences.

There are ____ quarters altogether.

___ quarters = ____ whole and ____ quarter.

Write sentences to describe these part-whole models.
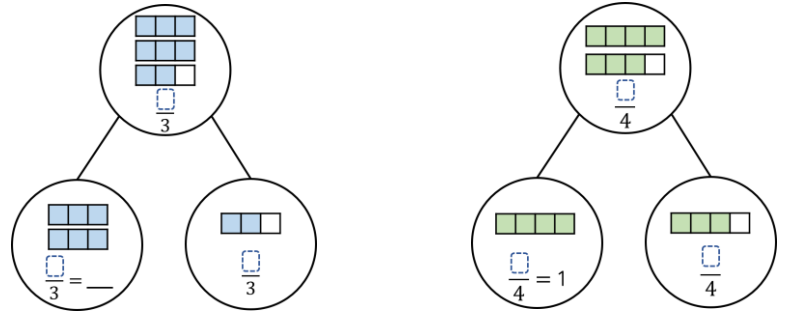
Complete. You may use part-whole models to help you.

$$\frac{10}{3} = \frac{9}{3} + \frac{\square}{3} = 3\frac{\square}{3}$$

$$\frac{\square}{3} = \frac{6}{3} + \frac{2}{3} = \square\frac{2}{3}$$

$$\frac{\square}{8} = \frac{16}{8} + \frac{3}{8} = \square\frac{\square}{\square}$$

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Fractions Greater than 1
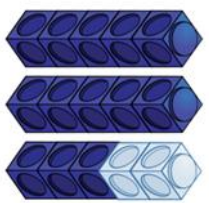
Reasoning and Problem Solving

R

3 friends share some pizzas.
Each pizza is cut into 8 equal slices.
Altogether, they eat 25 slices.
How many whole pizzas do they eat?

They eat 3 whole pizzas and 1 more slice.

Spot the mistake.

$\frac{13}{5}$ = 10 wholes and 3 fifths

There are 2 wholes not 10

$\frac{10}{5}$ = 2 wholes

$\frac{13}{5}$ = 2 wholes and 3 fifths

Rosie says,

$\frac{16}{4}$ is greater than $\frac{8}{2}$ because 16 is greater than 8

Do you agree?
Explain why.

I disagree with Rosie because both fractions are equivalent to 4

Children may choose to build both fractions using cubes, or draw bar models.

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Improper to Mixed Numbers

Notes and Guidance

Children convert improper fractions to mixed numbers for the first time. An improper fraction is a fraction where the numerator is greater than the denominator. A mixed number is a number consisting of an integer and a proper fraction.

It is important for children to see this process represented visually to allow them to make the connections between the concept and what happens in the abstract.

Mathematical Talk

How many parts are there in a whole?

What do you notice happens to the mixed number when the denominator increases and the numerator remains the same?

What happens when the numerator is a multiple of the denominator?

Varied Fluency

Whitney converts the improper fraction $\frac{14}{5}$ into a mixed number using cubes.
She groups the cubes into 5s, then has 4 left over.

$\frac{5}{5}$ is the same as ☐ $\frac{10}{5}$ is the same as ☐

$\frac{14}{5}$ as a mixed number is
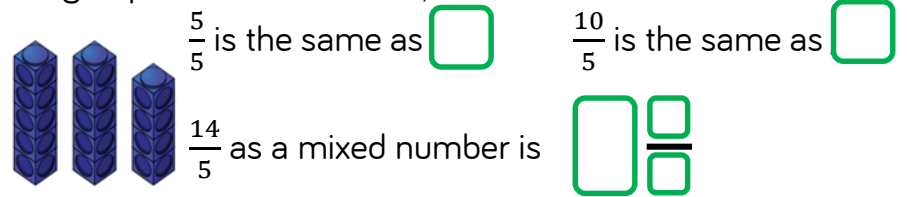
Use Whitney's method to convert $\frac{11}{3}, \frac{11}{4}, \frac{11}{5}$ and $\frac{11}{6}$

Tommy converts the improper fraction $\frac{27}{8}$ into a mixed number using bar models.

$$\frac{27}{8}$$

$$3$$

$$\frac{3}{8}$$

$$3\frac{3}{8}$$
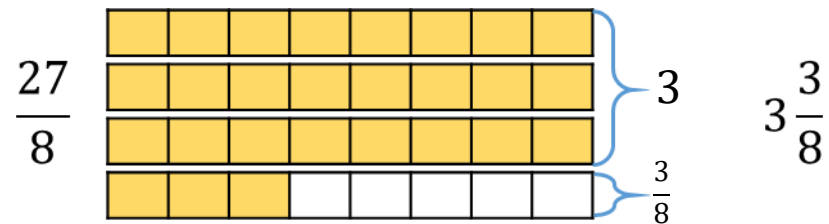
Use Tommy's method to convert $\frac{25}{8}, \frac{27}{6}, \frac{18}{7}$ and $\frac{32}{4}$

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Improper to Mixed Numbers

Reasoning and Problem Solving

Amir says,

$\frac{28}{3}$ is less than $\frac{37}{5}$ because 28 is less than 37

Do you agree?
Explain why.

Possible answer

I disagree because $\frac{28}{3}$ is equal to $9\frac{1}{3}$ and $\frac{37}{5}$ is equal to $7\frac{2}{5}$

$\frac{37}{5} < \frac{28}{3}$

Spot the mistake

- $\frac{27}{5} = 5\frac{1}{5}$

- $\frac{27}{3} = 8$

- $\frac{27}{4} = 5\frac{7}{4}$

- $\frac{27}{10} = 20\frac{7}{10}$

What mistakes have been made?

Can you find the correct answers?

Correct answers

- $5\frac{2}{5}$ (incorrect number of fifths)

- 9 (incorrect whole)

- $6\frac{3}{4}$ (still have an improper fraction)

- $2\frac{7}{10}$ (incorrect number of wholes)

Mixed Numbers to Improper

Notes and Guidance

Children now convert from mixed numbers to improper fractions using concrete and pictorial methods to understand the abstract method.

Ensure children always write their working alongside the concrete and pictorial representations so they can see the clear links to the abstract.

Mathematical Talk

How many quarters/halves/eighths/fifths are there in a whole?

How does multiplication support us in converting from mixed numbers to improper fractions?

Can you explain the steps in converting an improper fraction to a mixed number? Use the vocabulary: numerator, denominator, multiply, add

How could we use the previous bar model to help?

Varied Fluency

Whitney converts $3\frac{2}{5}$ into an improper fraction using cubes.

1 whole is equal to ☐ fifths.

3 wholes are equal to ☐ fifths.

☐ fifths + two fifths = ☐ fifths

Use Whitney's method to convert $2\frac{2}{3}$, $2\frac{2}{4}$, $2\frac{2}{5}$ and $2\frac{2}{6}$

Jack uses bar models to convert a mixed number into an improper fraction.
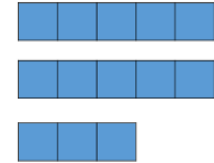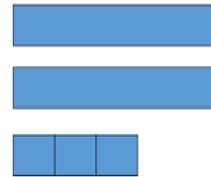
$2\frac{3}{5} =$ ☐ wholes + ☐ fifths

2 wholes = ☐ fifths

☐ fifths + ☐ fifths = ☐ fifths

Use Jack's method to convert $2\frac{1}{6}$, $4\frac{1}{6}$, $4\frac{1}{3}$ and $8\frac{2}{3}$

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Mixed Numbers to Improper

Reasoning and Problem Solving

Three children have incorrectly converted $3\frac{2}{5}$ into an improper fraction.

$$3\frac{2}{5} = \frac{6}{15}$$

Annie

$$3\frac{2}{5} = \frac{15}{5}$$

Mo

$$3\frac{2}{5} = \frac{32}{5}$$

Dexter

What mistake has each child made?

Annie has multiplied the numerator and denominator by 3

Mo has multiplied the correctly but then forgotten to add on the extra 2 parts.

Dexter has just placed 3 in front of the numerator.

Fill in the missing numbers.

How many different possibilities can you find for each equation?

$$2\frac{\square}{8} = \frac{\square}{8}$$

$$2\frac{\square}{5} = \frac{\square}{5}$$

Compare the number of possibilities you found.

$2\frac{1}{8} = \frac{17}{8}$ $2\frac{2}{8} = \frac{18}{8}$

$2\frac{3}{8} = \frac{19}{8}$ $2\frac{4}{8} = \frac{20}{8}$

$2\frac{5}{8} = \frac{21}{8}$ $2\frac{6}{8} = \frac{22}{8}$

$2\frac{7}{8} = \frac{23}{8}$

There will be 4 solutions for fifths.

Teacher notes: Encourage children to make generalisations that the number of solutions is one less than the denominator.

Number Sequences

Notes and Guidance

Children count up and down in a given fraction. They continue to use visual representations to help them explore number sequences.

Children also find missing fractions in a sequence and determine whether the sequence is increasing or decreasing and by how much.

Mathematical Talk

What are the intervals between the fractions?

Are the fractions increasing or decreasing?
How much are they increasing or decreasing by?

Can you convert the mixed numbers to improper fractions?
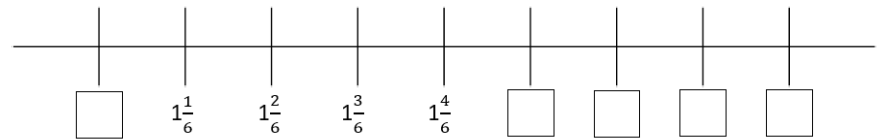Does this make it easier to continue the sequence?

Varied Fluency

▪ Use the counting stick to count up and down in these fractions.

- Start at 0 and count up in steps of $\frac{1}{4}$
- Start at 4 and count down in steps of $\frac{1}{3}$
- Start at 1 and count up in steps of $\frac{2}{3}$

▪ Complete the missing values on the number line.

$\square$ $1\frac{1}{6}$ $1\frac{2}{6}$ $1\frac{3}{6}$ $1\frac{4}{6}$ $\square$ $\square$ $\square$ $\square$

▪ Complete the sequences.

$\frac{3}{4}, \frac{\square}{\square}, 1\frac{3}{4}, 2\frac{1}{4}$ $\frac{\square}{\square}, 3\frac{1}{3}, \frac{\square}{\square}, 2\frac{2}{3}$

$\frac{\square}{\square}, 5\frac{1}{2}, 5\frac{7}{10}, 5\frac{9}{10}$ $\frac{3}{5}, \frac{\square}{\square}, \frac{\square}{\square}, 3$

Number Sequences

Reasoning and Problem Solving

Three children are counting in quarters.

Whitney

$\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}$

Teddy

$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}$

Eva

$\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{2}{4}, 1\frac{3}{4}$

Who is counting correctly?
Explain your reasons.

They are all correct, they are all counting in quarter.
Teddy has simplified all answers and Eva has converted improper fractions to mixed numbers.

Play the fraction game for four players.
Place the four fraction cards on the floor.
Each player stands in front of a fraction.
We are going to count up in tenths starting at 0
When you say a fraction, place your foot on your fraction.

$$\frac{1}{10} \qquad \frac{1}{5}$$

$$\frac{3}{10} \qquad \frac{1}{2}$$
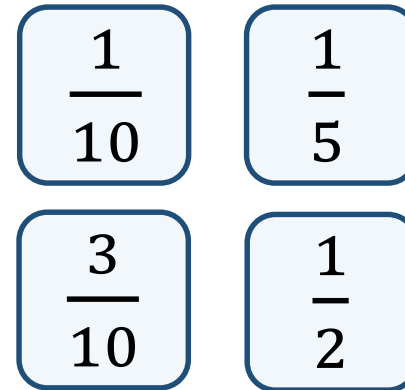
How can we make 4 tenths?
What is the highest fraction we can count to?
How about if we used two feet?

Children can make four tenths by stepping on one tenth and three tenths at the same time.
With one foot, they can count up to 11 tenths or one and one tenth.
With two feet they can count up to 22 tenths.

Compare & Order (Less than 1)

Notes and Guidance

Children build on their equivalent fraction knowledge to compare and order fractions less than 1 where the denominators are multiples of the same number.

Children compare the fractions by finding a common denominator or a common numerator. They use bar models to support their understanding.

Mathematical Talk

How does a bar model help us to visualise the fractions?
Should both of our bars be the same size? Why? What does this show us?
If the numerators are the same, how can we compare our fractions?
If the denominators are the same, how can we compare our fractions?
Do we always have to find a common denominator? Can we find a common numerator?

Varied Fluency

Use bar models to compare $\frac{5}{8}$ and $\frac{3}{4}$

$\square > \square$

$\square < \square$

Use this method to help you compare:

$\frac{5}{6}$ and $\frac{2}{3}$ $\frac{2}{3}$ and $\frac{5}{9}$ $\frac{7}{16}$ and $\frac{3}{8}$

Use common numerators to help you compare $\frac{2}{5}$ and $\frac{2}{3}$

$\square > \square$

$\square < \square$

Use this method to help you compare:

$\frac{6}{7}$ and $\frac{6}{8}$ $\frac{4}{9}$ and $\frac{4}{5}$ $\frac{4}{11}$ and $\frac{2}{5}$
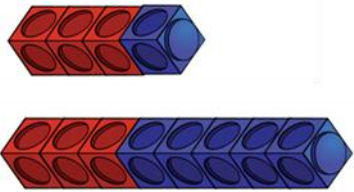
Order the fractions from greatest to smallest:

$\frac{3}{7}, \frac{3}{5}$ and $\frac{3}{8}$ $\frac{2}{3}, \frac{5}{6}$ and $\frac{7}{12}$ $\frac{6}{11}, \frac{3}{5}$ and $\frac{2}{3}$

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Compare & Order (Less than 1)
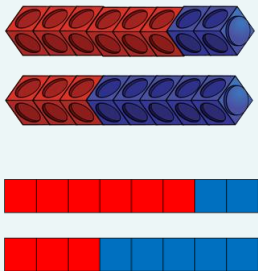
Reasoning and Problem Solving

Ron makes $\frac{3}{4}$ and $\frac{3}{8}$ out of cubes.

He thinks that $\frac{3}{8}$ is equal to $\frac{3}{4}$

Do you agree?
Explain your answer.

Possible answer:
I disagree with Ron because the two wholes are not equal. He could have compared using numerators or converted $\frac{3}{4}$ to $\frac{6}{8}$
If he does this he will see that $\frac{3}{4}$ is greater. Children may use bar models or cubes to show this.

Always, sometimes, never?

If one denominator is a multiple of the other you can simplify the fraction with the larger denominator to make the denominators the same.

Example:

Could $\frac{?}{4}$ and $\frac{?}{12}$ be simplified to $\frac{?}{4}$ and $\frac{?}{4}$?

Prove it.

Sometimes

It does not work for some fractions e.g. $\frac{8}{15}$ and $\frac{3}{5}$

But does work for others e.g. $\frac{1}{4}$ and $\frac{9}{12}$

Compare & Order (More than 1)

Notes and Guidance

Children use their knowledge of ordering fractions less than 1 to help them compare and order fractions greater than 1

They use their knowledge of common denominators to help them.

Children will compare both improper fractions and mixed numbers during this step.

Mathematical Talk

How can we represent the fractions?

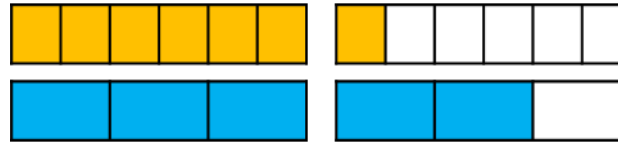How does the bar help us see which fraction is the greatest?

Can we use our knowledge of multiples to help us?

Can you predict which fractions will be greatest? Explain how you know.

Is it more efficient to compare using numerators or denominators?

Varied Fluency
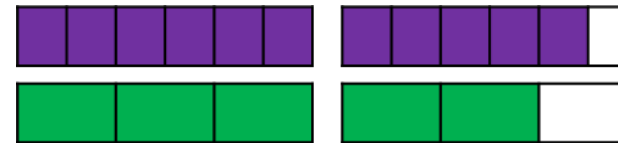
Use bar models to compare $\frac{7}{6}$ and $\frac{5}{3}$

$\square > \square$

$\square < \square$

Use this method to help you compare:

$\frac{5}{2}$ and $\frac{9}{4}$ $\frac{11}{6}$ and $\frac{5}{3}$ $\frac{9}{4}$ and $\frac{17}{8}$

Use a bar model to compare $1\frac{2}{3}$ and $1\frac{5}{6}$

$\square > \square$

$\square < \square$

Use this method to help you compare:

$1\frac{3}{4}$ and $1\frac{3}{8}$ $1\frac{5}{8}$ and $1\frac{1}{2}$ $2\frac{3}{7}$ and $2\frac{9}{14}$

Order the fractions from greatest to smallest using common denominators:

$\frac{8}{5}$, $\frac{11}{10}$ and $\frac{17}{20}$ $1\frac{2}{3}$, $1\frac{7}{24}$ and $\frac{11}{12}$
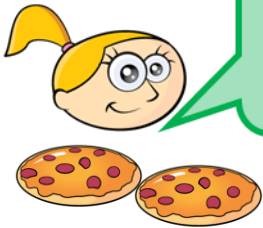
$\frac{?}{20}$, $\frac{?}{20}$ and $\frac{?}{20}$

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Compare & Order (More than 1)

Reasoning and Problem Solving

Eva and Alex each have two identical pizzas.

Eva says,

> I have cut each pizza into 6 equal pieces and eaten 8

Alex says,
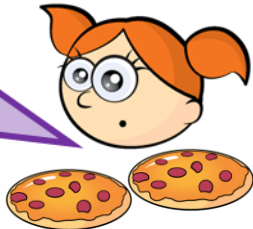
> I have cut each pizza into 9 equal pieces and eaten 15

Who ate the most pizza?

Use a drawing to support your answer.

Alex ate the most pizza because $\frac{15}{9}$ is greater than $\frac{8}{6}$

Dora looks at the fractions $1\frac{7}{12}$ and $1\frac{3}{4}$

She says,

> $1\frac{7}{12}$ is greater than $1\frac{3}{4}$ because the numerator is larger

Do you agree?

Explain why using a model.

Possible answer:
I do not agree because $1\frac{3}{4}$ is equivalent to $1\frac{9}{12}$ and this is greater than $1\frac{7}{12}$

Add & Subtract Fractions

Notes and Guidance

Children recap their Year 4 understanding of adding and subtracting fractions with the same denominator.

They use bar models to support understanding of adding and subtracting fractions.

Mathematical Talk

How many equal parts do I need to split my bar into?

Can you convert the improper fraction into a mixed number?

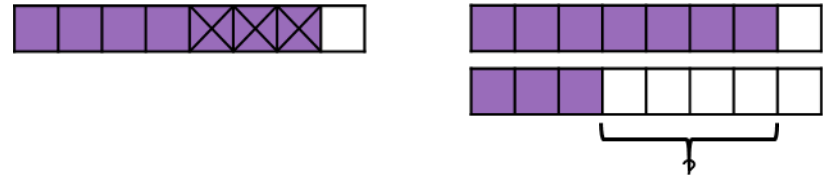How can a bar model help you balance both sides of the equals sign?

Varied Fluency

Here is a bar model to calculate $\frac{3}{5} + \frac{4}{5}$

$$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$$

Use a bar model to solve the calculations:

$$\frac{3}{8} + \frac{3}{8} \qquad \frac{5}{6} + \frac{1}{6} \qquad \frac{5}{3} + \frac{5}{3}$$

Here are two bar models to calculate $\frac{7}{8} - \frac{3}{8}$

What is the difference between the two methods?
Use your preferred method to calculate:

$$\frac{5}{8} - \frac{1}{8} \qquad \frac{9}{7} - \frac{4}{7} \qquad \frac{5}{3} - \frac{5}{3} \qquad 1 - \frac{2}{5}$$

Calculate:

$$\frac{3}{7} + \frac{5}{7} = \frac{\square}{\square} + \frac{4}{7} \qquad \frac{9}{5} - \frac{5}{5} = \frac{6}{5} - \frac{\square}{\square} \qquad \frac{2}{3} + \frac{\square}{\square} = \frac{11}{3} - \frac{4}{3}$$

Add & Subtract Fractions

Reasoning and Problem Solving

How many different ways can you balance the equation?

$$\frac{5}{9} + \frac{\square}{9} = \frac{8}{9} + \frac{\square}{9}$$

Possible answers:

$$\frac{5}{9} + \frac{3}{9} = \frac{8}{9} + \frac{0}{9}$$

$$\frac{5}{9} + \frac{4}{9} = \frac{8}{9} + \frac{1}{9}$$

$$\frac{5}{9} + \frac{5}{9} = \frac{8}{9} + \frac{2}{9}$$

Any combination of fractions where the numerators add up to the same total on each side of the equals sign.

A chocolate bar has 12 equal pieces.

Amir eats $\frac{5}{12}$ more of the bar than Whitney.

There is one twelfth of the bar remaining.

What fraction of the bar does Amir eat?

What fraction of the bar does Whitney eat?

Amir eats $\frac{8}{12}$ of the chocolate bar and

Whitney eats $\frac{3}{12}$ of the chocolate bar.

Add Fractions within 1

Notes and Guidance

Children add fractions with different denominators for the first time where one denominator is a multiple of the other.

They use pictorial representations to convert the fractions so they have the same denominator.

Ensure children always write their working alongside the pictorial representations so they see the clear links.

Mathematical Talk

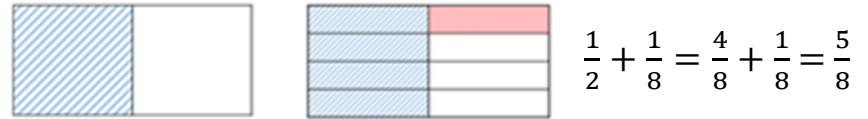Can you find a common denominator? Do you need to convert both fractions or just one?

Can you explain Mo and Rosie's methods to a partner? Which method do you prefer?

How do Mo and Rosie's methods support finding a common denominator?

Varied Fluency

Mo is calculating $\frac{1}{2} + \frac{1}{8}$

He uses a diagram to represent the sum.

$$\frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$$

Use Mo's method to solve :

$$\frac{1}{2} + \frac{3}{8} \qquad \frac{1}{4} + \frac{3}{8} \qquad \frac{7}{10} + \frac{1}{5}$$

Rosie is using a bar model to solve $\frac{1}{4} + \frac{3}{8}$
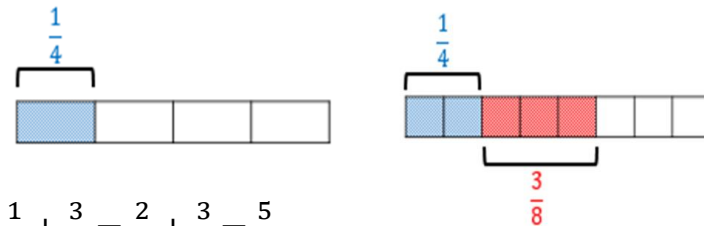
$$\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$$

Use a bar model to solve:

$$\frac{1}{6} + \frac{5}{12} \qquad \frac{2}{9} + \frac{1}{3} \qquad \frac{1}{3} + \frac{4}{15}$$

Add Fractions within 1
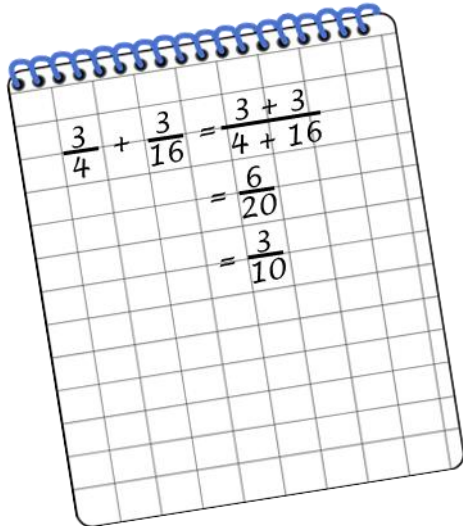
Reasoning and Problem Solving

$$\frac{5}{16} + \frac{\boxed{}}{8} = \frac{15}{16}$$

$$\frac{\boxed{}}{20} + \frac{7}{10} = \frac{17}{20}$$

5

3

Annie solved this calculation.

$$\frac{3}{4} + \frac{3}{16} = \frac{3+3}{4+16}$$
$$= \frac{6}{20}$$
$$= \frac{3}{10}$$

Can you spot and explain her mistake?

Annie is wrong because she has just added the numerators and the denominators. When adding fractions with different denominators you need to find a common denominator.

Two children are solving $\frac{1}{3} + \frac{4}{15}$

Eva starts by drawing this model:

Alex starts by drawing this model:

Can you explain each person's method and how they would complete the question?
Which method do you prefer and why?

Possible answer:
Each child may have started with a different fraction in the calculation.
e.g. Eva has started by shading a third. She now needs to divide each third into five equal parts so there are fifteen equal parts altogether. Eva will then shade $\frac{4}{15}$ and will have $\frac{9}{15}$ altogether.

Add 3 or More Fractions

Notes and Guidance

Children add more than 2 fractions where two denominators are a multiple of the other.

They use a bar model to continue exploring this.

Ensure children always write their working alongside the pictorial representations so they see the clear links.

Mathematical Talk

Can you find a common denominator? Do you need to convert both fractions or just one?

Can you explain Ron's method to a partner? How does Ron's method support finding a common denominator?

Can you draw what Farmer Staneff's field could look like? What fractions could you divide your field into?

Why would a bar model not be efficient for this question?

Varied Fluency

Ron uses a bar model to calculate $\frac{2}{5} + \frac{1}{10} + \frac{3}{20}$

$$\frac{2}{5} \qquad \frac{4}{10} \quad \frac{1}{10} \qquad \frac{8}{20} \quad \frac{2}{20} \quad \frac{3}{20}$$
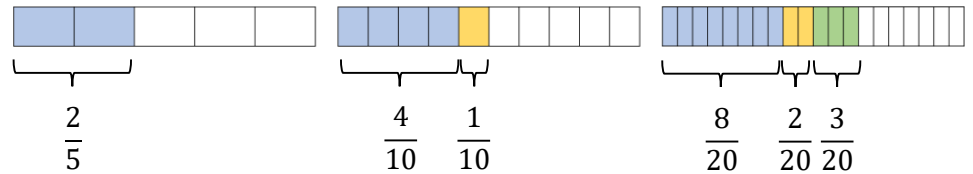
Use a bar model to solve:

$$\frac{1}{4} + \frac{3}{8} + \frac{5}{16} \qquad \frac{1}{2} + \frac{1}{6} + \frac{1}{12}$$

Farmer Staneff owns a field.

He plants carrots on $\frac{1}{3}$ of the field.

He plants potatoes on $\frac{2}{9}$ of the field.

He plants onions on $\frac{5}{18}$ of the field.

What fraction of the field is covered altogether?

Complete the fractions.

$$\frac{1}{5} + \frac{\square}{10} + \frac{8}{20} = 1 \qquad \frac{1}{5} + \frac{\square}{15} + \frac{1}{30} = 1$$

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White Rose Maths

Add 3 or More Fractions

Reasoning and Problem Solving

Eva is attempting to answer:

$$\frac{3}{5} + \frac{1}{10} + \frac{3}{20}$$

$$\frac{3}{5} + \frac{1}{10} + \frac{3}{20} = \frac{7}{35}$$

Do you agree with Eva?
Explain why.

Eva is wrong because she has added the numerators and denominators together and hasn't found a common denominator. The correct answer is $\frac{17}{20}$

Jack has added 3 fractions together to get an answer of $\frac{17}{18}$

What 3 fractions could he have added?

Can you find more than one answer?

Possible answers:

$$\frac{1}{18} + \frac{4}{18} + \frac{13}{18}$$

$$\frac{1}{9} + \frac{5}{9} + \frac{5}{18}$$

$$\frac{1}{6} + \frac{5}{9} + \frac{2}{9}$$

$$\frac{1}{18} + \frac{1}{6} + \frac{13}{18}$$

$$\frac{1}{3} + \frac{1}{6} + \frac{4}{9}$$

Add Fractions

Notes and Guidance

Children continue to represent adding fractions using pictorial methods to explore adding two or more proper fractions where the total is greater than 1

Children can record their totals as an improper fraction but will then convert this to a mixed number using their prior knowledge.

Mathematical Talk

How does the pictorial method support me to add the fractions?

Which common denominator will we use?

How do my times-tables support me to add fractions?

Which representation do you prefer? Why?

Varied Fluency

Step 1	Step 2	Step 3

$$\frac{1}{3} + \frac{5}{6} + \frac{5}{12} = 1\frac{7}{12}$$

Explain each step of the calculation.
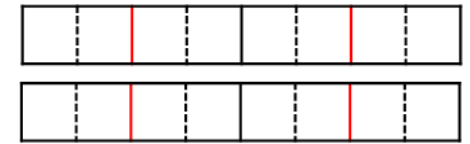Use this method to help you add the fractions.
Give your answer as a mixed number.

$$\frac{2}{3} + \frac{1}{6} + \frac{7}{12} \qquad \frac{1}{4} + \frac{7}{8} + \frac{3}{16} \qquad \frac{1}{2} + \frac{5}{6} + \frac{5}{12}$$

Use the bar model to add the fractions. Record your answer as a mixed number.

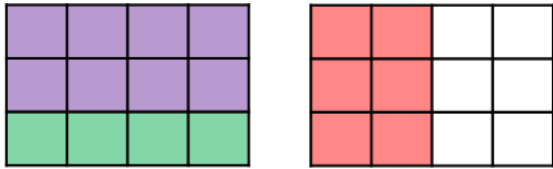$$\frac{3}{4} + \frac{3}{8} + \frac{1}{2} =$$

Draw your own models to solve:

$$\frac{5}{12} + \frac{1}{6} + \frac{1}{2} \qquad \frac{11}{20} + \frac{3}{5} + \frac{1}{10} \qquad \frac{3}{4} + \frac{5}{12} + \frac{1}{2}$$

Add Fractions

Reasoning and Problem Solving

Annie is adding three fractions.
She uses the model to help her.

What could her three fractions be?

How many different combinations can you find?

Can you write a number story to represent your calculation?

Possible answer:

$$\frac{2}{3} + \frac{4}{12} + \frac{1}{2} = 1\frac{1}{2}$$

Other equivalent fractions may be used.

Example story: Some children are eating pizzas. Jack eats two thirds, Amir eats four twelfths and Dexter eats half a pizza. How much pizza did they eat altogether?

The sum of three fractions is $2\frac{1}{8}$

The fractions have different denominators.

All of the fractions are greater than or equal to a half.

None of the fractions are improper fractions.

All of the denominators are factors of 8

What could the fractions be?

$$\frac{1}{2} + \frac{3}{4} + \frac{7}{8}$$

Children could be given less clues and explore other possible solutions.

Add Mixed Numbers

Notes and Guidance

Children move on to adding two fractions where one or both are mixed numbers or improper fractions.

They will use a method of adding the wholes and then adding the parts. Children will record their answer in its simplest form.

Children can still draw models to represent adding fractions.

Mathematical Talk

How can we partition these mixed numbers into whole numbers and fractions?

What will the wholes total? Can I add the fractions straight away?

What will these mixed numbers be as improper fractions?

If I have an improper fraction in the question, should I change it to a mixed number first? Why?

Varied Fluency

$$1\frac{1}{3} + 2\frac{1}{6} = 3 + \frac{3}{6} = 3\frac{3}{6} \text{ or } 3\frac{1}{2}$$

$$1 + 2 = 3$$

$$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6}$$

Add the fractions by adding the whole first and then the fractions. Give your answer in its simplest form.

$$3\frac{1}{4} + 2\frac{3}{8} \qquad 4\frac{1}{9} + 3\frac{2}{3} \qquad 2\frac{5}{12} + 2\frac{1}{3}$$

$$1\frac{3}{4} + 2\frac{1}{8} = \frac{7}{4} + \frac{17}{8} = \frac{14}{8} + \frac{17}{8} = \frac{31}{8} = 3\frac{7}{8}$$

Add the fractions by converting them to improper fractions.

$$1\frac{1}{4} + 2\frac{5}{12} \qquad 2\frac{1}{9} + 1\frac{1}{3} \qquad 2\frac{1}{6} + 2\frac{2}{3}$$

Add these fractions.

$$4\frac{7}{9} + 2\frac{1}{3} \qquad \frac{17}{6} + 1\frac{1}{3} \qquad \frac{15}{8} + 2\frac{1}{4}$$

How do they differ from previous examples?

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Add Mixed Numbers

Reasoning and Problem Solving

Jack and Whitney have some juice.

Jack drinks $2\frac{1}{4}$ litres and Whitney drinks $2\frac{5}{12}$ litres.

How much do they drink altogether?

Complete this using two different methods.

Which method do you think is more efficient? Why?

They drink $4\frac{2}{3}$ litres altogether.

Encourage children to justify which method they prefer and why. Ensure children discuss which method is more or less efficient.

Fill in the missing numbers.

$$4\frac{5}{6} + \boxed{}\frac{\boxed{}}{\boxed{}} = 10\frac{1}{3}$$

$5\frac{3}{6}$ or $5\frac{1}{2}$

Subtract Fractions

Notes and Guidance

Children subtract fractions with different denominators for the first time, where one denominator is a multiple of the other.

It is important that subtraction is explored as both take away and finding the difference.
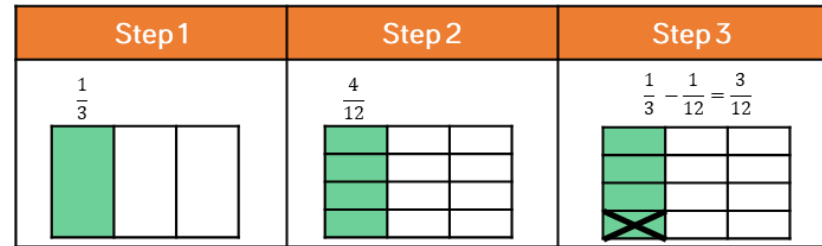
Mathematical Talk

What could the common denominator be?

Can you draw a model to help you solve the problem?

Is it easier to use a take away bar model (single bar model) or a bar model to find the difference (comparison model)?

Varied Fluency

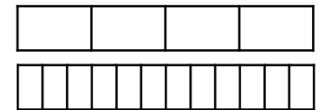| Step 1	Step 2	Step 3
$\frac{1}{3}$	$\frac{4}{12}$	$\frac{1}{3} - \frac{1}{12} = \frac{3}{12}$

Explain each step of the calculation.
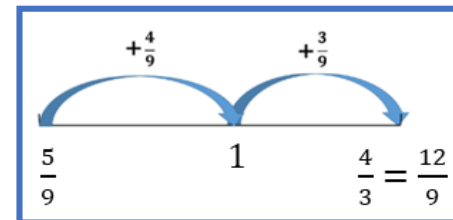Use this method to help you solve $\frac{5}{6} - \frac{1}{3}$ and $\frac{7}{8} - \frac{5}{16}$

Tommy and Teddy both have the same sized chocolate bar. Tommy has $\frac{3}{4}$ left, Teddy has $\frac{5}{12}$ left.
How much more does Tommy have?

Amir uses a number line to find the difference between $\frac{5}{9}$ and $\frac{4}{3}$

$+\frac{4}{9}$ $+\frac{3}{9}$

$\frac{5}{9}$ 1 $\frac{4}{3} = \frac{12}{9}$

Use this method to find the difference between:

$\frac{3}{4}$ and $\frac{5}{12}$ $\frac{19}{15}$ and $\frac{3}{5}$ $\frac{20}{9}$ and $\frac{4}{3}$

Subtract Fractions

Reasoning and Problem Solving

Which subtraction is the odd one out?

A $\dfrac{13}{4} - \dfrac{3}{8}$

B $\dfrac{10}{3} - \dfrac{2}{9}$

C $\dfrac{23}{7} - \dfrac{1}{3}$

Explain why.

Possible answers:

C is the odd one out because the denominators aren't multiples of each other.

A is the odd one out because the denominators are even.

B is the odd one out because it is the only answer above 3

The perimeter of the rectangle is $\dfrac{16}{9}$

$\dfrac{2}{3}$

?

Work out the missing length.

The missing length is $\dfrac{2}{9}$

Subtract Mixed Numbers (1)

Notes and Guidance

Children apply their understanding of subtracting fractions where one denominator is a multiple of the other to subtract proper fractions from mixed numbers.

They continue to use models and number lines to support their understanding.
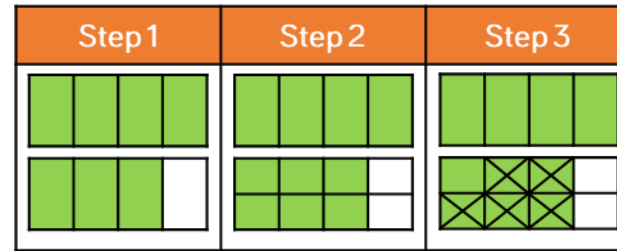
Mathematical Talk

Which fraction is the greatest? How do you know?

If the denominators are different, what can we do?

Can you simplify your answer?

Which method do you prefer when subtracting fractions: taking away or finding the difference?

Varied Fluency

Step 1	Step 2	Step 3

$$1\frac{3}{4} - \frac{5}{8} = 1\frac{1}{8}$$

Use this method to help you solve:
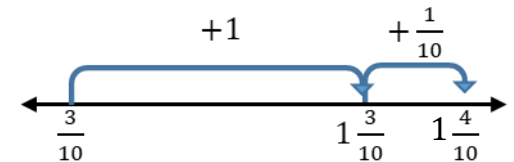
$$2\frac{3}{5} - \frac{3}{10} \qquad 1\frac{2}{3} - \frac{1}{6} \qquad 1\frac{5}{6} - \frac{7}{12}$$

Use a number line to find the difference between $1\frac{2}{5}$ and $\frac{3}{10}$

$$1\frac{2}{5} = 1\frac{4}{10}$$

$$+1 \qquad +\frac{1}{10}$$

$$\frac{3}{10} \qquad 1\frac{3}{10} \qquad 1\frac{4}{10}$$

Use a number line to find the difference between:

$$3\frac{5}{6} \text{ and } \frac{1}{12} \qquad 5\frac{5}{7} \text{ and } \frac{3}{14} \qquad 2\frac{7}{9} \text{ and } \frac{11}{18}$$

Solve:

$$1\frac{2}{3} - \frac{5}{6} \qquad 1\frac{3}{4} - \frac{7}{8} \qquad 2\frac{3}{8} - \frac{11}{16}$$

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Subtract Mixed Numbers (1)

Reasoning and Problem Solving

Amir is attempting to solve $2\frac{5}{14} - \frac{2}{7}$
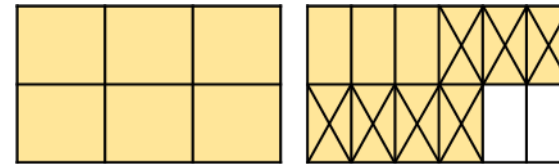
Here is his working out:

$$2\frac{5}{14} - \frac{2}{7} = 2\frac{3}{7}$$

Do you agree with Amir?
Explain your answer.

Possible answer:

Amir is wrong because he hasn't found a common denominator when subtracting the fractions he has just subtracted the numerators and the denominators. The correct answer is $2\frac{1}{14}$

Here is Rosie's method.
What is the calculation?

Can you find more than one answer?
Why is there more than one answer?

The calculation could be $1\frac{5}{6} - \frac{7}{12}$ or $1\frac{10}{12} - \frac{7}{12}$

There is more than one answer because five sixths and ten twelfths are equivalent. Children should be encouraged to write the question as $1\frac{5}{6} - \frac{7}{12}$ so that all fractions are in their simplest form.

White Rose Maths

Subtract Mixed Numbers (2)

Notes and Guidance

Children use prior knowledge of fractions to subtract two fractions where one is a mixed number and you need to break one of the wholes up.

They use the method of flexible partitioning to create a new mixed number so they can complete the calculation.
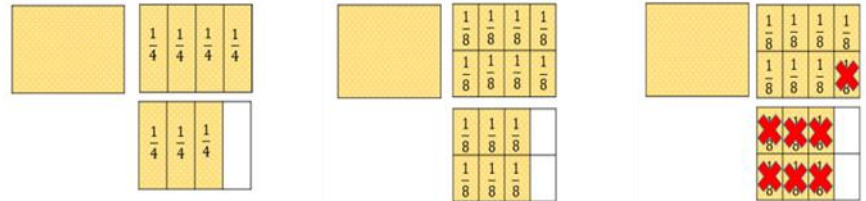
Mathematical Talk

Is flexible partitioning easier than converting the mixed number to an improper fraction?

Do we always have to partition the mixed number?

When can we subtract a fraction without partitioning the mixed number in a different way?

Varied Fluency

We can work out $2\frac{3}{4} - \frac{7}{8}$ using this method.

Use this method to calculate:

$$3\frac{1}{3} - \frac{5}{6} \qquad 4\frac{1}{5} - \frac{7}{10} \qquad 5\frac{2}{3} - \frac{4}{9}$$

Use flexible partitioning to solve $7\frac{1}{3} - \frac{5}{6}$

$$7\frac{1}{3} - \frac{5}{6} = 6 + 1\frac{1}{3} - \frac{5}{6} = 6 + 1\frac{2}{6} - \frac{5}{6} = 6\frac{3}{6} = 6\frac{1}{2}$$

Use this method to calculate:

$$4\frac{2}{3} - \frac{5}{6} \qquad 4\frac{1}{5} - \frac{7}{15} \qquad 5\frac{1}{4} - \frac{7}{8}$$

Mr Brown has $3\frac{1}{4}$ bags of flour. He uses $\frac{7}{8}$ of a bag. How much flour does he have left?

White Rose Maths

Subtract Mixed Numbers (2)

Reasoning and Problem Solving

Place 2, 3 and 4 in the boxes to make the calculation correct.

$$27\frac{1}{\square} - \frac{\square}{6} = 26\frac{\square}{3}$$

$$27\frac{1}{3} - \frac{4}{6} = 26\frac{2}{3}$$

3 children are working out $6\frac{2}{3} - \frac{5}{6}$

They partition the mixed number in the following ways to help them.

Dora $\quad 5 + 1\frac{2}{3} - \frac{5}{6}$

Alex $\quad 5 + 1\frac{4}{6} - \frac{5}{6}$

Jack $\quad 5 + \frac{10}{6} - \frac{5}{6}$

Are they all correct?
Which method do you prefer?
Explain why.

All three children are correct.

$1\frac{2}{3}, 1\frac{4}{6}$ and $\frac{10}{6}$ are all equivalent therefore all three methods will help children to correctly calculate the answer.

White Rose Maths

Subtract 2 Mixed Numbers

Notes and Guidance

Children use different strategies to subtract two mixed numbers.

Building on learning in previous steps, they look at partitioning the mixed numbers into wholes and parts and build on their understanding of flexible partitioning as well as converting to improper fractions when an exchange is involved.

Mathematical Talk

Why is subtracting the wholes and parts separately easier with some fractions than others?

Can you show the subtraction as a difference on a number line? Bar model? How are these different to taking away?

Does making the whole numbers larger make the subtraction any more difficult? Explain why.

Varied Fluency

Here is a bar model to calculate $3\frac{5}{8} - 2\frac{1}{4}$

$\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

$3\frac{5}{8} - 2\frac{1}{4} = 1\frac{3}{8}$
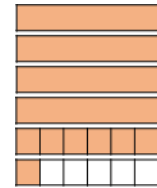
$3 - 2 = 1$
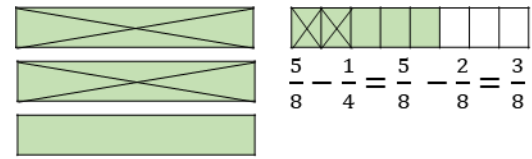
Use this method to calculate:

$3\frac{7}{8} - 2\frac{3}{4}$ $5\frac{5}{6} - 2\frac{1}{3}$ $3\frac{8}{9} - 2\frac{5}{27}$

Why does this method not work effectively for $5\frac{1}{6} - 2\frac{1}{3}$?

Here is a method to calculate $5\frac{1}{6} - 2\frac{1}{3}$

$5\frac{1}{6} - 2\frac{1}{3} = 4\frac{7}{6} - 2\frac{1}{3} = 4\frac{7}{6} - 2\frac{2}{6} = 2\frac{5}{6}$

Use this method to calculate:

$3\frac{1}{4} - 2\frac{5}{8}$ $5\frac{1}{3} - 2\frac{7}{12}$ $27\frac{1}{3} - 14\frac{7}{15}$

Subtract 2 Mixed Numbers

Reasoning and Problem Solving

There are three colours of dog biscuits in a bag of dog food: red, brown and orange.

The total mass of the dog food is 7 kg.

The mass of red biscuits is $3\frac{3}{4}$ kg and the mass of the brown biscuits is $1\frac{7}{16}$ kg.

What is the mass of orange biscuits?

$3\frac{3}{4} + 1\frac{7}{16} = 5\frac{3}{16}$

$7 - 5\frac{3}{16} = 1\frac{13}{16}$

The mass of orange biscuits is $1\frac{13}{16}$ kg.

Rosie has $20\frac{3}{4}$ cm of ribbon.

Annie has $6\frac{7}{8}$ cm less ribbon than Rosie.

How much ribbon does Annie have?

How much ribbon do they have altogether?

Annie has $13\frac{7}{8}$ cm of ribbon.

Altogether they have $34\frac{5}{8}$ cm of ribbon.

Multiply by an Integer (1)

Notes and Guidance

Children are introduced to multiplying fractions by a whole number for the first time. They link this to repeated addition and see that the denominator remains the same, whilst the numerator is multiplied by the integer.
This is shown clearly through the range of models to build the children's conceptual understanding of multiplying fractions. Children should be encouraged to simplify fractions where possible.

Mathematical Talk

How is multiplying fractions similar to adding fractions?

What is the same/different between: $\frac{3}{4} \times 2$ and $2 \times \frac{3}{4}$?

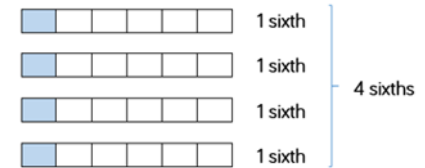Which bar model do you find the most useful?

Which bar model helps us to convert from an improper fraction to a mixed number most effectively?

What has happened to the numerator/denominator?

Varied Fluency

■ Work out $\frac{1}{6} \times 4$ by counting in sixths.

$$\frac{1}{6} \times 4 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$
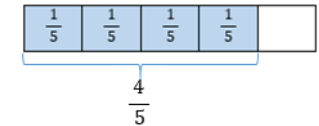
Use this method to work out:

$2 \times \frac{1}{3}$ $\qquad$ $\frac{1}{5} \times 3$ $\qquad$ $6 \times \frac{1}{10}$

■ Mo uses a single bar model to work out: $\frac{1}{5} \times 4 = \frac{4}{5}$

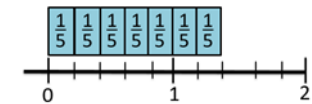Use this method to work out:

$\frac{1}{4} \times 3$ $\qquad$ $6 \times \frac{1}{8}$ $\qquad$ $\frac{1}{10} \times 8$

■ Eva uses a number line and repeated addition to work out $\frac{1}{5} \times 7 = \frac{7}{5} = 1\frac{2}{5}$

Use this method to work out:

$5 \times \frac{1}{8}$ $\qquad$ $\frac{1}{3} \times 3$ $\qquad$ $\frac{1}{4} \times 7$

Multiply by an Integer (1)

Reasoning and Problem Solving

Amir is multiplying fractions by a whole number.

$$\frac{1}{5} \times 5 = \frac{5}{25}$$

Can you explain his mistake?

Amir has multiplied both the numerator and the denominator so he has found an equivalent fraction. Encourage children to draw models to represent this correctly.

Always, sometimes, never?

When you multiply a unit fraction by the same number as it's denominator the answer will be one whole.

Always - because the numerator was 1 it will always be the same as your denominator when multiplied which means that it is a whole.

e.g. $\frac{1}{3} \times 3 = \frac{3}{3} = 1$

I am thinking of a unit fraction.

When I multiply it by 4 it will be equivalent to $\frac{1}{2}$

When I multiply it by 2 it will be equivalent to $\frac{1}{4}$

What is my fraction?

What do I need to multiply my fraction by so that my answer is equivalent to $\frac{3}{4}$?

Can you create your own version of this problem?

$\frac{1}{8}$ because

$4 \times \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

and

$2 \times \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$

6 because

$6 \times \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$

Multiply by an Integer (2)

Notes and Guidance

Children apply prior knowledge of multiplying a unit fraction by a whole number to multiplying a non-unit fraction by a whole number.

They use similar models and discuss which method will be the most efficient depending on the questions asked.

Reinforce the concept of commutativity by showing examples of the fraction first and the integer first in the multiplication.

Mathematical Talk

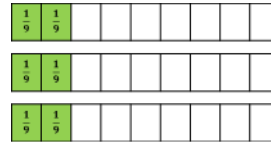Can you show me 3 lots of $\frac{3}{10}$ on a bar model?

How many tenths do we have altogether?

How does repeated addition help us with this multiplication?

How does a number line help us see the multiplication?

Varied Fluency
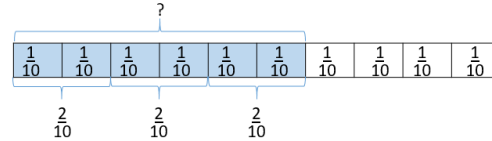
Count the number of ninths to work $3 \times \frac{2}{9}$

Use this method to work out:

$\frac{3}{8} \times 2$ $\qquad$ $\frac{5}{16} \times 3$ $\qquad$ $4 \times \frac{2}{11}$

Use the model to help you solve $3 \times \frac{2}{10}$
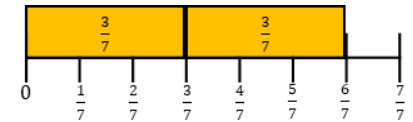
Use this method to work out:

$\frac{2}{7} \times 3$ $\qquad$ $\frac{3}{16} \times 4$ $\qquad$ $4 \times \frac{5}{12}$

Use the number line to help you solve $2 \times \frac{3}{7}$

Use this method to work out:

$\frac{3}{10} \times 3$ $\qquad$ $\frac{2}{7} \times 2$ $\qquad$ $4 \times \frac{3}{20}$

Multiply by an Integer (2)

Reasoning and Problem Solving

Use the digit cards only once to complete these multiplications.

9 2 4 6 3

$$\boxed{} \times \dfrac{\boxed{}}{\boxed{}} = \dfrac{\boxed{}}{\boxed{}}$$

1 2 3 4 5 6

$$\boxed{} \times \dfrac{\boxed{}}{\boxed{}} = \dfrac{\boxed{}}{\boxed{}}$$
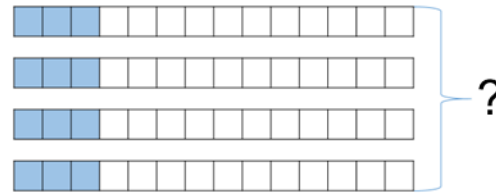
$$2 \times \dfrac{3}{4} = \dfrac{9}{6}$$

Possible answers:

$$2 \times \dfrac{1}{3} = \dfrac{4}{6}$$

$$2 \times \dfrac{1}{4} = \dfrac{3}{6}$$

Whitney has calculated $4 \times \dfrac{3}{14}$

?

From the picture I can see that $4 \times \dfrac{3}{14} = \dfrac{12}{56}$

Do you agree?

Explain why.

Possible answer:

I disagree. Whitney has shaded 12 fourteenths. She has counted all of the boxes to give her the denominator when it is not needed. The answer should be $\dfrac{12}{14}$ or $\dfrac{6}{7}$

Multiply by an Integer (3)

Notes and Guidance

Children use their knowledge of fractions to multiply a mixed number by a whole number.

They use the method of repeated addition, multiplying the whole and part separately and the method of converting to an improper fraction then multiplying.

Continue to explore visual representations such as the bar model.

Mathematical Talk

How could you represent this mixed number?

What is the denominator? How do you know?

How many wholes are there? How many parts are there?

What is multiplying fractions similar to? (repeated addition)

What representation could you use to convert a mixed number to an improper fraction?

Varied Fluency

Use repeated addition to work out $2\frac{2}{3} \times 4$

$$2\frac{2}{3} \times 4 = 2\frac{2}{3} + 2\frac{2}{3} + 2\frac{2}{3} + 2\frac{2}{3} = 8\frac{8}{3} = 10\frac{2}{3}$$

Use this method to solve:

$2\frac{1}{6} \times 3 \qquad 1\frac{3}{7} \times 2 \qquad 3\frac{1}{3} \times 4$

Partition your fraction to help you solve $2\frac{3}{4} \times 3$

$2 \times 3 = 6$

$\frac{3}{4} \times 3 = \frac{9}{4} = 2\frac{1}{4}$

$6 + 2\frac{1}{4} = 8\frac{1}{4}$

Use this method to answer:

$2\frac{5}{6} \times 3 \qquad 3\frac{4}{7} \times 2 \qquad 2\frac{1}{3} \times 5$

$$1\frac{5}{6} \times 3 = \frac{11}{6} \times 3 = \frac{33}{6} = 5\frac{3}{6} = 5\frac{1}{2}$$

Convert to an improper fraction to calculate:

$3\frac{2}{7} \times 4 \qquad 2\frac{4}{9} \times 2 \qquad 4 \times 3\frac{3}{5}$

Multiply by an Integer (3)

Reasoning and Problem Solving

Jack runs $2\frac{2}{3}$ miles three times per week.

Dexter runs $3\frac{3}{4}$ miles twice a week.

Who runs the furthest during the week?

Explain your answer.

Jack runs $2\frac{2}{3} \times 3 = 8$ miles.

Dexter runs $3\frac{3}{4} \times 2 = 7\frac{1}{2}$ miles.

Jack runs further by half a mile.

Work out the missing numbers.

$$2\frac{\square}{8} \times \square = 7\frac{7}{8}$$

Explain how you worked it out.

Possible answer:

$$2\frac{5}{8} \times 3 = 7\frac{7}{8}$$

I knew that the multiplier could not be 4 because that would give an answer of at least 8. So the multiplier had to be 3. That meant that the missing numerator had to give a product of 15. I knew that 5 multiplied by 3 would give 15

Fractions of a Quantity

Notes and Guidance

Children use their knowledge of finding unit fractions of a quantity, to find non-unit fractions of a quantity.

They use concrete and pictorial representations to support their understanding. Children link bar modelling to the abstract method in order to understand why the method works.

Mathematical Talk

What is the whole? What fraction of the whole are we finding? How many equal parts will I divide the whole into?

What's the same and what's different about the calculations? Can you notice a pattern?

What fraction of her chocolate bar does Whitney have left? How many grams does she have left? Can you represent this on a bar model?

Varied Fluency

R

Mo has 12 apples.
Use counters to represent his apples and find:

$\frac{1}{2}$ of 12 $\frac{1}{4}$ of 12 $\frac{1}{3}$ of 12 $\frac{1}{6}$ of 12

Now calculate:

$\frac{2}{2}$ of 12 $\frac{3}{4}$ of 12 $\frac{2}{3}$ of 12 $\frac{5}{6}$ of 12

What do you notice? What's the same and what's different?

Use a bar model to help you represent and find:

$\frac{1}{7}$ of 56 = 56 ÷ ☐

56

$\frac{2}{7}$ of 56 $\frac{3}{7}$ of 56 $\frac{4}{7}$ of 56 $\frac{4}{7}$ of 28 $\frac{7}{7}$ of 28

Whitney eats $\frac{3}{8}$ of 240 g bar of chocolate.
How many grams of chocolate has she eaten?

46

Fractions of a Quantity

Reasoning and Problem Solving

R

True or False?

> To find $\frac{3}{8}$ of a number, divide by 3 and multiply by 8

Convince me.

False.
Divide the whole by 8 to find one eighth and then multiply by three to find three eighths of a number.

Ron gives $\frac{2}{9}$ of a bag of 54 marbles to Alex.

Teddy gives $\frac{3}{4}$ of a bag of marbles to Alex.

Ron gives Alex more marbles than Teddy.

How many marbles could Teddy have to begin with?

$$\frac{2}{9} \text{ of } 54 > \frac{3}{4} \text{ of } \boxed{}$$

Teddy could have 16, 12, 8 or 4 marbles to begin with.

Fraction of an Amount

Notes and Guidance

Children recap previous learning surrounding finding unit and non-unit fractions of amounts, quantities and measures.

It is important that the concept is explored pictorially through bar models to support children to make sense of the abstract.

Mathematical Talk

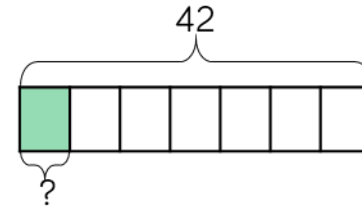How many equal groups have you shared 49 into? Why?

What does each equal part represent as a fraction and an amount?

What could you do to 1 metre to make the calculation easier?

1 litre = _____ ml   1 kg = _____ g

Varied Fluency

Find $\frac{1}{7}$ of 42

42

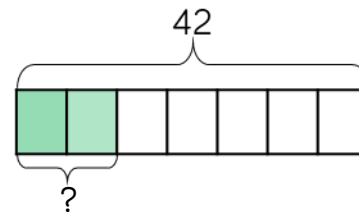$42 \div 7 = 6$
$\frac{1}{7}$ of 42 is 6

?

Use this method to find:

$\frac{1}{8}$ of 56 $\frac{1}{6}$ of 480 $\frac{1}{9}$ of 81 m

Find $\frac{2}{7}$ of 42

42

$42 \div 7 = 6$
$6 \times 2 = 12$
$\frac{2}{7}$ of 42 is 12

?

Use this method to find:

$\frac{3}{8}$ of 56 $\frac{5}{6}$ of 480 $\frac{4}{9}$ of 81 m

Draw a bar model to help you calculate:

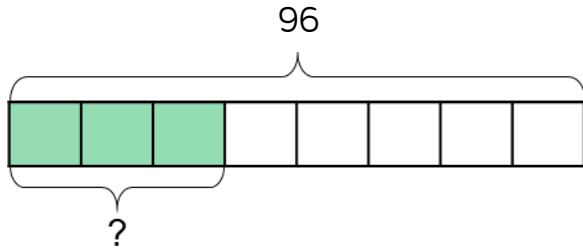$\frac{4}{5}$ of 1 m $\frac{5}{12}$ of 1.44 litres $\frac{3}{7}$ of 21 kg

Fraction of an Amount

Reasoning and Problem Solving

Write a problem that matches the bar model.

96

?

What other questions could you ask from this model?

$\frac{7}{16}$ of a class are boys.

There are 18 girls in the class.

How many children are in the class?

Possible response:
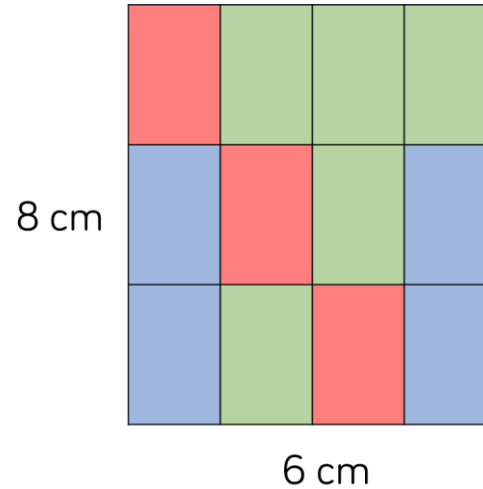
There are 96 cars in a car park. $\frac{3}{8}$ of them are red. How many cars are red?

How many were not red? etc.

There are 32 children in the class.

Find the area of each colour in the rectangle.

8 cm

6 cm

What would happen if one of the red or green rectangles was changed to a blue?

Area of rectangle:
$6 \times 8 = 48 \text{ cm}^2$

Blue
$\frac{4}{12}$ of $48 = 16 \text{ cm}^2$

Red
$\frac{3}{12}$ of $48 = 12 \text{ cm}^2$

Green
$\frac{5}{12}$ of $48 = 20 \text{ cm}^2$

Children need to show that this would impact both the blue and the other colour.
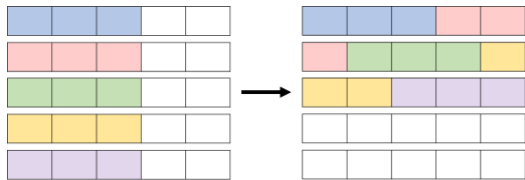
Fractions as Operators

Notes and Guidance

Children link their understanding of fractions of amounts and multiplying fractions to use fractions as operators.

They use their knowledge of commutativity to help them understand that you can change the order of multiplication without changing the product.

Mathematical Talk

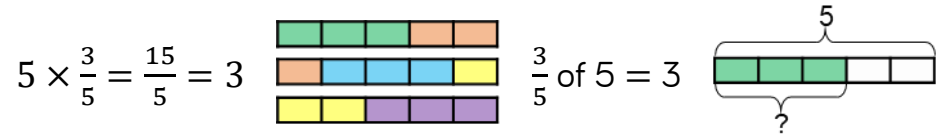What is the same and different about these bar models?

Is it easier to multiply a fraction or find a fraction of an amount?

Does it depend on the whole number you are multiplying by?

Can you see the link between the numbers?

Varied Fluency

Tommy has calculated and drawn a bar model for two calculations.

$$5 \times \frac{3}{5} = \frac{15}{5} = 3$$

$$\frac{3}{5} \text{ of } 5 = 3$$

What's the same and what's different about Tommy's calculations?

Complete:

2 lots of $\frac{1}{10} = \Box$ $\frac{1}{10}$ of $2 = \Box$

6 lots of $\Box = 3$ $\Box$ of $6 = 3$

8 lots of $\frac{1}{4} = \Box$ $\frac{1}{4}$ of $8 = \Box$

> Which calculation on each row is easier? Why?

Use this to complete:

$$20 \times \frac{4}{5} = \frac{\Box}{\Box} \text{ of } 20 = \Box$$

$$\Box \times \frac{2}{3} = \frac{\Box}{\Box} \text{ of } 18 = 12$$

$$\Box \times \frac{1}{3} = \frac{1}{3} \text{ of } \Box = 20$$

Fractions as Operators

Reasoning and Problem Solving

Which method would you use to complete these calculations: multiply the fractions or find the fraction of an amount?

Explain your choice for each one. Compare your method to your partner.

$$25 \times \frac{3}{5} \text{ or } \frac{3}{5} \text{ of } 25$$

$$6 \times \frac{2}{3} \text{ or } \frac{2}{3} \text{ of } 6$$

$$5 \times \frac{3}{8} \text{ or } \frac{3}{8} \text{ of } 5$$

Possible response:

1. Children may find it easier to find 3 fifths of 25 rather than multiply 25 by 3
2. Children may choose either as they are of similar efficiency.
3. Children will probably find it more efficient to multiply than divide 5 by 8

Dexter and Jack are thinking of a two-digit number between 20 and 30

Dexter finds two thirds of the number.

Jack multiplies the number by $\frac{2}{3}$

Their new two-digit number has a digit total that is one more than that of their original number.

What number did they start with?

Show each step of their calculation.

They started with 24

Dexter:
$24 \div 3 = 8$
$8 \times 2 = 16$

Jack:
$24 \times 2 = 48$
$48 \div 3 = 16$